Math Review Preparation for Placement Test
Wayne Community College
Algebra

The mathematics department at Wayne Community College has developed a review to help refresh basic algebra skills before taking the placement test. The purpose of the placement test is to determine the appropriate course(s) students need to take. In this review, important concepts are summarized. Examples illustrating these concepts are also included. Practice with these examples as a guide. Do not be concerned if you are unable to work any or all of the problems.

There are several parts to the Asset Placement Test. It is divided into the following sections:

A) Numerical Skills: 25 minutes/30 questions
B) Elementary Algebra: 25 minutes/25 questions
C) Intermediate Algebra: 25 minutes/25 questions
D) College Algebra: 25 minutes/25 questions

Preparation:

- Review this review information.
- Arrive at least 10 minutes prior to testing.
- Bring at least two #2 pencils to the test.
- Be aware of the time limits.

Taking the Test

- Read all directions carefully.
- Read each question carefully paying attention to phrases such as "all of the above" and "none of the above".
- Don't forget any parts of the test.
- Always guess on every question you do not know. There is no penalty for guessing.
- Check answers in you have time.

Wayne Community College developed this review with input from Coastal Carolina Community College as well as Lenoir Community College. Good luck on your placement test!

Concept #1: Exponents

Positive integer exponent: This number indicates how many times the base is to be multiplied.

Example:

$$3^4 = 3 \cdot 3 \cdot 3 \cdot 3 = 81$$

$$(-1)^5 = (-1)(-1)(-1)(-1)(-1) = -1$$

Negative integer exponent: If this number is applied to a base, it is equal to the reciprocal of the base raised to the opposite exponent.

Example:

$$2^{-3} = \frac{1}{2^3} = \frac{1}{(2)(2)(2)} = \frac{1}{8}$$

$$x^{-5} = \frac{1}{x^5}$$

Zero exponent: If this is applied to any base (except 0), the resulting answer will be 1.

Example:

$$5^0 = 1$$

$$x^0 = 1$$

$$(-2)^0 = 1$$

Fractional Exponent: This indicates that a radical should be applied to the base. The numerator of the exponent denotes the power to which the base is raised, and the denominator of the exponent denotes the root to be taken.

Example:

$$8^{\frac{2}{3}} = \sqrt[3]{8^2} = \sqrt[3]{64} = 4$$

$$or$$

$$8^{\frac{2}{3}} = (\sqrt[3]{8})^2 = 2^2 = 4$$

Moving Factors: A factor can be moved from the numerator of a fraction to the denominator (or vice versa) by changing the sign of the exponent.

Example:

$\frac{x^4 y^{-3}}{z^5} = \frac{x^4}{y^3 z^5}$ Notice how the "y" factor was moved to the denominator by changing the -3 to a positive 3.

More examples of exponents:

$$(\frac{2}{3})^3 = \frac{2^3}{3^3} = \frac{8}{27}$$

and

$$(x+1)^2 = (x+1)(x+1) = x^2 + x + x + 1 = x^2 + 2x + 1$$

and

$$(3xy)^3 = 3^3 x^3 y^3 = 27x^3 y^3$$

and

$$x^2 \cdot x^5 = x^7 (add\_\exp onents\_with\_like\_bases)$$

and

$$\frac{x^8}{x^3} = x^5 (subtract\_\exp onents\_with\_like\_bases)$$

Add or Subtract Expressions:

- Combine like terms. (Keep the same exponent.)

Examples:

$$3x^2 + 5x^2 = 8x^2$$

or

$$4x^2 - 3x - 6x^2$$
$$= 4x^2 - 6x^2 - 3x$$
$$= -2x^2 - 3x$$

Concept #2: Operations on Polynomials

Add Polynomials:

- Remove parentheses
- Combine like terms

Example:

$$(4x^3 - 2x^2 - 7x) + (-6x^3 - 3x^2 + 5)$$
$$4x^3 - 2x^2 - 7x + -6x^3 - 3x^2 + 5$$
$$4x^3 - 6x^3 - 2x^2 - 3x^2 - 7x + 5$$
$$-2x^3 - 5x^2 - 7x + 5$$

Subtract Polynomials:

- Distribute the negative. This will change the signs of each term in the 2nd polynomial.
- Combine like terms.

Example:

$$(4x^3 - 2x^2 - 7x) - (-6x^3 - 3x^2 + 5)$$
$$4x^3 - 2x^2 - 7x + 6x^3 + 3x^2 - 5$$
$$4x^3 + 6x^3 - 2x^2 + 3x^2 - 7x - 5$$
$$10x^3 + x^2 - 7x - 5$$

Multiply Polynomials:

- Multiply each term of one polynomial by each term of the other polynomial.
- Combine like terms.
- When multiplying two binomials, often referred to as FOIL, First, Outer, Inner, and Last.

Examples:

$$1.(2x^2 - 7x + 1)(3x + 4)$$
$$(2x^2)(3x) + (2x^2)(4) - (7x)(3x) - (7x)(4) + (1)(3x) + (1)(4)$$
$$6x^3 + 8x^2 - 21x^2 - 28x + 3x + 4$$
$$6x^3 - 13x^2 - 25x + 4$$

$$2.(x + 3)(x - 2)$$
$$x^2 - 2x + 3x - 6$$
$$x^2 + x - 6$$

Divide Polynomials:

- To divide a polynomial by a monomial, divide the monomial into each term of the polynomial.

Example:

$$\frac{3x^2+6x+10}{2x}$$

$$\frac{3x^2}{2x}+\frac{6x}{2x}+\frac{10}{2x}$$

$$\frac{3x}{2}+3+\frac{5}{x}$$

- To divide a polynomial by a polynomial, use long division.
- If either polynomial has a "missing term", use zero as a placeholder.
- Divide the first term of expression by the 1st term of divisor.
- Multiply the result by each term.
- Subtract by changing the sign and combining like terms.
- Bring down the next term in the dividend.
- Repeat above steps until process is complete.
- Add to the resulting quotient the remainder divided by the divisor.

Concept #3: Inequalities

Solving linear inequalities:

- Use the same technique for solving equations, but remember to reverse the symbol when multiplying or dividing by a negative number.

Example:

$$-2x < 20$$
$$x > -10$$

Solving compound inequalities:

- Isolate the variable in the center by performing the same operation on all three parts.

Example:

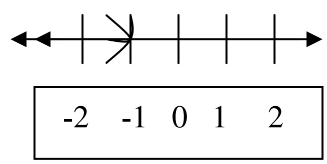
$$-13 \le 6x - 1 \le 3$$
$$-12 \le 6x \le 4$$
$$\frac{-12}{6} \le \frac{6x}{6} \le \frac{4}{6}$$
$$-2 \le x \le \frac{2}{3}$$

<u>Graphing inequalities:</u>

- Isolate the variable.
- Shade in the numbers on the real number line that satisfy the inequality.
- Use brackets to show the endpoint is included with $\le or \ge$. Use parentheses the show the endpoint is NOT included with < or >.

Example: Graph:

$$x + 2 < 1$$
$$x < -1$$



Note: Parenthesis was used to denote -1 is NOT an included solution.

<u>Ways to write the solution of an inequality.</u>

- Interval Notation: Solve the inequality.
- List smallest number and largest number in the solution set separated by a comma.
- If no smallest number, use $(\infty$
- If no largest number, use ∞)
- Use brackets with $\le or \ge$
- Use parentheses with < or >

Example: Write $-6 \le x < 3$ using interval notation.
[-6, 3)

Write $x < 2$ using interval notation.
$(-\infty,$ 2)

Concept #4: Factoring

Rules for factoring:

- Factor out the greatest common factor (GCF). Divide each term by the largest expression that will divide into every term.)
- Use the factoring technique that corresponds to the number of terms.
- Two Terms: Use difference of squares, or the difference or sum of cubes formulas.

  Difference of squares: $a^2 - b^2 = (a+b)(a-b)$

  Difference of cubes: $a^3 - b^3 = (a^2 + ab + b^2)$

  Sum of cubes: $a^3 + b^3 = (a^2 - ab + b^2)$
- Three terms: Use the trial-and-error technique.
- Four terms: Use the grouping technique.
- Check factoring by multiplying the answer.

Examples: Factor each.

$$1.)8x^3 - 50x = 2x(4x^2 - 25) = 2x(2x+5)(2x-5)$$

$$2.)3x^3 - 24 = 3(x^3 - 8) = 3(x-2)(x^2 + 2x + 4)$$

$$3.)5x^2 - 15x + 10 = 5(x^2 - 3x + 2) = 5(x-2)(x-1)$$

$$4.)x^3 - 2x^2 + 4x - 8 = x^2(x-2) + 4(x-2) = (x^2 + 4)(x-2)$$

Concept #5: Rational Expressions

Simplifying Rational Expressions:

- Factor the numerator and denominator.
- Cancel common factors.

Example:

$$\frac{x^2 + 2x - 15}{3x - 9} = \frac{(x+5)(x-3)}{3(x-3)} = \frac{x+5}{3}$$

Multiplying or Dividing Rational Expressions:

- Completely factor numerator and denominator.
- Perform the indicated operation.
- Cancel common factors.

Example:

$$\frac{2x}{3x-12} \div \frac{x^2-2x}{x^2-6x+8} = \frac{2x}{3(x-4)} \cdot \frac{(x-4)(x-2)}{x(x-2)} = \frac{2x(x-4)(x-2)}{3x(x-4)(x-2)} = \frac{2}{3}$$

Example:

$$\frac{x}{5x^2-20x} \cdot \frac{x-4}{2x^2+x-3} = \frac{x(x-4)}{5x(x-4)(2x+3)(x-1)} = \frac{1}{5(2x+3)(x-1)}$$

Adding or Subtracting Rational Expressions:

- Completely factor the denominator.
- Find the least common denominator by using each factor represented, raised to the highest power occurring on each factor.
- Multiply numerator and denominator by an expression resulting in the common denominator.
- Perform the operation and simplify.

Example:

$$\frac{6x}{x^2-4} - \frac{3}{(x-2)^2}$$

$$= \frac{6x}{(x-2)(x+2)} - \frac{3}{(x-2)(x-2)}$$

$$= \frac{6x(x-2)}{(x+2)(x-2)^2} - \frac{3(x+2)}{(x+2)(x-2)^2}$$

$$= \frac{6x^2-12x-3x-6}{(x+2)(x-2)^2}$$

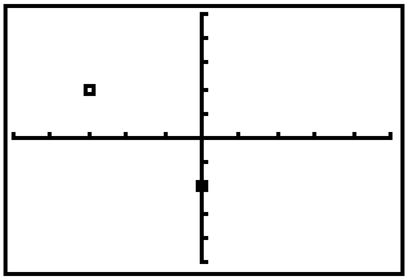
$$= \frac{6x^2-15x-6}{(x+2)(x-2)^2}$$

Concept #6: Graphing

Points:

- An ordered pair can represent a point on the rectangular coordinate system. The first coordinate gives the position along the horizontal axis, and the second gives the vertical position.

Example: Plot (0, -2) and (-3, 2).



Distance between two points:

- To find the distance between two given points $(x_1, y_1) and (x_1, y_1)$, use the following formula: $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

Example: Find the distance between (3, -2) and (5, 1).

$$\sqrt{(5-3)^2 + (1-(-2))^2}$$
$$= \sqrt{(2)^2 + (3)^2}$$
$$= \sqrt{4+9} = \sqrt{13}$$

Midpoint between two points:

- To find the midpoint between two given points $(x_1, y_1) and (x_1, y_1)$, use the following formula: $\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$

Example: Find the midpoint between (3, -2) and (5, 1).

$$\left(\frac{3+5}{2} + \frac{-2+1}{2}\right)$$
$$= \left(\frac{8}{2}, \frac{-1}{2}\right) = \left(4, \frac{-1}{2}\right)$$

Equations: The graph of an equation is the plotted set of all ordered pairs whose coordinates satisfy the equation.

Finding x-intercepts and y-intercepts:

- To find the x-intercepts of an equation, set y = 0 and solve for x.
- To find the y-intercepts of an equation, set x = 0 and solve for y.

Example: Find the x and y intercepts of the graph 3x + y = 6.

x-intercept	y-intercept
(let y = 0)	(let x = 0)

3x + 0 = 6   3(0) + y = 6

3x = 6   y = 6

x = 2

Therefore, the x-intercept is (2, 0) and the y-intercept is (0,6).

Sketching graphs:

- Solve for y.
- Make a table of values showing several solution points.
- Plot the points in a rectangular coordinate system.
- Connect the points with a smooth curve or line.

Example: Sketch the graph of $y - x^2 = 3$.

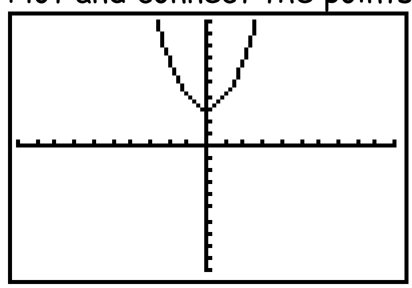
Solve for y: $y = x^2 + 3$.

Make a table:

X	Y1	
-3	12	
-2	7	
-1	4	
0	3	
1	4	
2	7	
3	12	

X= -3

Plot and connect the points.



Lines:

Slope: Slope is denoted by the letter m. To find the slope of the line through two given points $(x_1, y_1) and (x_1, y_1)$, use the following formula: $m = \frac{y_2 - y_1}{x_2 - x_1}$.

Example: Find the slope of the line through (-3, 7) and (3, 1).

$$m = \frac{7-1}{-3-3} = \frac{6}{-6} = -1$$

Writing the equation of a line: To find the equation of a line with given characteristics, use the following formula.

Point Slope Formula: $y - y_1 = m(x - x_1)$.

Example: Find the equation of the line through (-3, 7) and (3, 1).

From the example above, the slope was determined to be –1.

Now substitute the proper numbers into the Point Slope Formula: (Choose either of the given points.)

$$y-7=-1(x-(-3))$$
$$y-7=-1(x+3)$$
$$y-7=-x-3$$
$$y=-x+4$$

<u>Slope Intercept Form:</u> y = mx + b is the graph of a line. Slope is denoted by the "m" and the y-intercept is "b".

Example: y = -x + 4 is the equation of a line with slope = -1 and the y-intercept of (0, 4).

<u>Vertical Lines:</u>

- x = a is a vertical line through (a, 0)
- Vertical lines have undefined slope.

<u>Horizontal Lines:</u>

- y = b is a horizontal line through (0, b)
- Horizontal lines have zero slope.

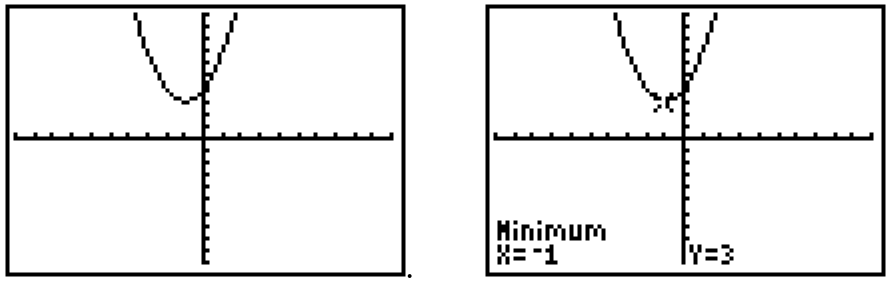
<u>Parallel and Perpendicular Lines:</u>

- Parallel lines have equal slopes.
- Perpendicular lines have opposite reciprocal slopes. Or, the product of their slopes is equal to –1.

<u>Parabolas:</u>

- The graph of $y=ax^2+bx+c$ where $a\neq 0$ is a parabola with vertex $\left(\frac{-b}{2a},\left(f\frac{-b}{2a}\right)\right)$.
- If a is positive, the parabola opens upward.
- If a is negative, the parabola opens downward.

Example: The graph of $f(x)=x^2+2x+4$ is a parabola with opens upward and has a vertex (-1, 3).



From the above graph, the vertex is obviously (-1, 3).

Concept #7: Simplifying Radicals

Simplifying radicals:

- Remove all possible factors from the radical.
- Write the number as a product using the largest factor that is a perfect Kth power where K is the index.
- Write the variable factors as a product using the largest exponent that is a multiple of K.
- Apply the radical to each part of the fraction or product. The roots are written outside the radical and the "leftover" factors remain under the radical.

Example: Simplify $\sqrt{50}$.

$$\sqrt{50} = \sqrt{25 \cdot 2} = \sqrt{25} \cdot \sqrt{2} = 5\sqrt{2}$$

Example: Simplify $\sqrt[3]{54x^6y^8}$.

$$\sqrt[3]{54x^6y^8} = \sqrt[3]{27 \cdot 2x^6y^6y^2} = 3x^2y^2\sqrt[3]{2y^2}$$

Rationalize Denominators:

- To rationalize a denominator with one term, multiply the numerator and denominator by a radical that will produce a perfect Kth power radicand in the denominator and simplify.

Example: Rationalize the denominator.

$$\frac{1}{\sqrt[3]{x^2}} \cdot \frac{\sqrt[3]{x}}{\sqrt[3]{x}} = \frac{\sqrt[3]{x}}{\sqrt[3]{x^3}} = \frac{\sqrt[3]{x}}{x}$$

- To rationalize a denominator with two terms, multiply the numerator and denominator by the conjugate of the denominator (opposite middle sign), then multiply by using FOIL and simplify.

Example: Rationalize the denominator.

$$\frac{2}{3-\sqrt{5}} = \frac{2}{3-\sqrt{5}} \cdot \frac{3+\sqrt{5}}{3+\sqrt{5}} = \frac{2(3+\sqrt{5})}{9+3\sqrt{5}-3\sqrt{5}-5} = \frac{2(3+\sqrt{5})}{4}$$

- To reduce the index of a radical, rewrite using a fractional exponent and reduce the fraction before converting back to radical notation.

Example: Simplify.

$$\sqrt[6]{x^2} = x^{\frac{2}{6}} = x^{\frac{1}{3}} = \sqrt[3]{x^1}$$

- To reduce the index of a radical if the fraction cannot be reduced, try writing the radicand using an exponent.

Example: Simplify.

$$\sqrt[4]{9} = 9^{\frac{1}{4}} = (3^2)^{\frac{1}{4}} = 3^{\frac{2}{4}} = 3^{\frac{1}{2}} = \sqrt{3}$$

- Note: $\sqrt{-1} = i$, an imaginary number.

Example: Simplify.

$$\sqrt{-8} = \sqrt{-4 \cdot 2} = \sqrt{-1} \cdot \sqrt{4} \cdot \sqrt{2} = i2\sqrt{2} = 2i\sqrt{2}$$

- To add or subtract radicals (combine like radicals), simplify each radical, and then combine those having the same index and radicand by adding/subtracting their coefficients.

Example: Add.

$$\sqrt{75} + \sqrt{27}$$
$$= \sqrt{25 \cdot 3} + \sqrt{9 \cdot 3}$$
$$= 5\sqrt{3} + 3\sqrt{3}$$
$$= 8\sqrt{3}$$

- To multiply radicals with the same indices, multiply the radicands.

Example: Multiply.

$$\sqrt{5}\cdot\sqrt{2}=\sqrt{5\cdot 2}=\sqrt{10}$$

- To divide radicals with the same indices, divide the radicands.

Example: Divide.

$$\frac{\sqrt{27}}{\sqrt{3}}=\sqrt{\frac{27}{3}}=\sqrt{9}=3$$

- To multiply and divide radicals with different indices:
- Write each radical with fractional exponents.
- Rewrite each with a common denominator.
- Convert to the radical form.
- Multiply or divide as usual.

Example: Multiply.

$$\sqrt{x}\cdot\sqrt[3]{x}=x^{\frac{1}{2}}\cdot x^{\frac{1}{3}}=x^{\frac{3}{6}}\cdot x^{\frac{2}{6}}=x^{\frac{5}{6}}=\sqrt[6]{x^5}$$

Concept #8: Solving Special Equations

Quadratic Equations: $ax^2+bx+c=0$

- If the equation has the form $ax^2+c=$(no x term), isolate the squared quantity and extract square roots. Don't forget the $\pm$.
- If the expression ax^2+bx+c will factor, then factor, set each factor equal to zero, and solve each equation.
- The Quadratic Formula can be used on any quadratic equation. Set one side equal to zero, identify a, b, and c and substitute into the formula. If $ax^2+bx+c=0$, then $x=\dfrac{-b\pm\sqrt{b^2-4ac}}{2a}$.

Examples: Solve each.

$$x^2 - 5 = 0$$
$$x^2 = 5$$
$$x = \pm\sqrt{5}$$

$$2x^2 + 9x = 5$$
$$2x^2 + 9x - 5 = 0$$
$$(2x-1)(x+5) = 0$$
$$2x - 1 = 0 \quad or \quad x + 5 = 0$$
$$x = \frac{1}{2} \quad or \quad x = -5$$

$$x^2 + 7x + 4 = 0$$
$$x = \frac{-7 \pm \sqrt{49 - 4(1)(4)}}{2(1)}$$
$$x = \frac{-7 \pm \sqrt{33}}{2}$$

Radical Equations:

- Isolate the most complicated radical on one side.
- Raise each side to the power equal to the index of the radical.
- If the radical remains, repeat above steps.
- Solve the resulting equation.
- A check is necessary if the original equation involves a radical with an even index.

Example: Solve.

$$\sqrt{x+2} + 4 = 10$$
$$\sqrt{x+2} = 6$$
$$\left(\sqrt{x+2}\right)^2 = 6^2$$ Remember to check by substituting into original.
$$x + 2 = 36$$
$$x = 34$$

Higher Order Factorable Equations:

- Set one side equal to zero.
- Factor.
- Set each factor equal to zero and solve each simpler equation.

Example: Solve.

$$x^3 = 4x$$
$$x^3 - 4x = 0$$
$$x(x^2 - 4) = 0$$
$$x(x-2)(x+2) = 0$$
$$x = 0, 2, -2$$

Absolute Value Equations:

- Isolate the absolute value expression.
- Set the expression inside the absolute value equal to the constant and also equal to its negative.
- Solve each equation and check the solution.

Example: Solve.

$$3|x-7|+9=12$$

$$3|x-7|=3$$

$$|x-7|=1$$

$$x-7=1 or x-7=-1$$

$$x=8,6$$